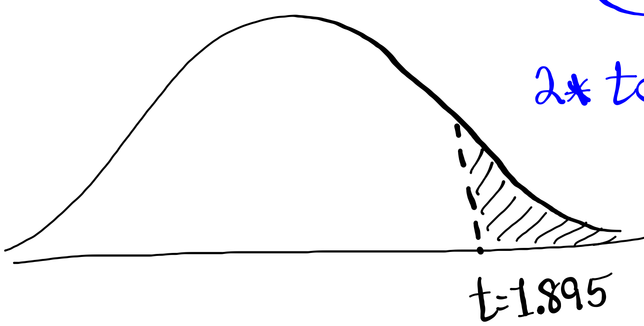
Statistics

Summer 2021

Lecture 14

(SG 22)

find twice the shaded area below with df=12.

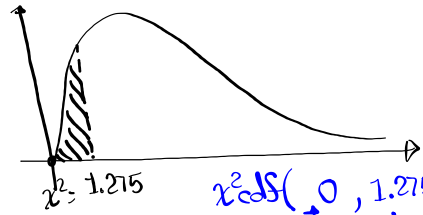


2* tcdf(1.895, E99, 12)

↑ lower ↑ upper

= .082

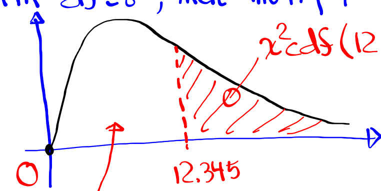
Find the shaded area below with $df = 10$.



$\chi^2cdf(0, 1.275, 10) = \boxed{5.2 \times 10^{-4}}$

(0 = lower, 1.275 = upper, 10 = df)

Find the area on each side of $\chi^2 = 12.345$ with $df = 8$, then multiply the smaller area by 2.



$\chi^2cdf(12.345, E99, 8) = \boxed{.136}$

$\chi^2cdf(0, 12.345, 8) = \boxed{.864}$

2 * Smaller area

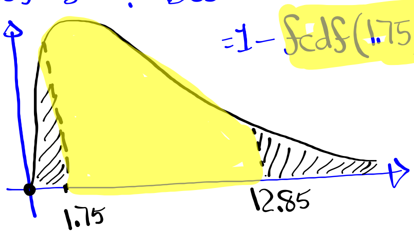
$= 2(.136)$

$= \boxed{.272}$

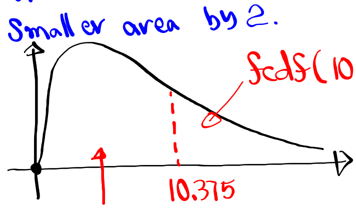
Find $P(F < 1.75 \text{ or } F > 12.85)$ with $Ndf = 5$ & $Ddf = 35$.

$= 1 - Fcdf(1.75, 12.85, 5, 35)$

$= \boxed{.851}$



Find the area of each side of $F = 10.375$ with $Ndf = 4$ & $Ddf = 28$, then multiply the smaller area by 2.



$Fcdf(10.375, E99, 4, 28) = \boxed{2.8 \times 10^{-5}}$

$2(2.8 \times 10^{-5})$

$= \boxed{5.6 \times 10^{-5}}$

$Fcdf(0, 10.375, 4, 28) = \boxed{.9999}$

≈ 1

what is degrees of freedom?

Non-Statistical Explanation

14 Students, I bring 14 donuts. 1 for each

First student ⇒ 14 choices
2nd " ⇒ 13 "
3rd " ⇒ 12 "
⋮
Last " ⇒ No choice

$df = 14 - 1 = \boxed{13}$

You have 7 clean shirts.

Monday → 7 choices
Tuesday → 6 "
Wednesday → 5 "
Saturday 2 choies
Sunday No choice

$df = 7 - 1 = \boxed{6}$

$\alpha \to$ Alpha

$0 < \alpha < 1$

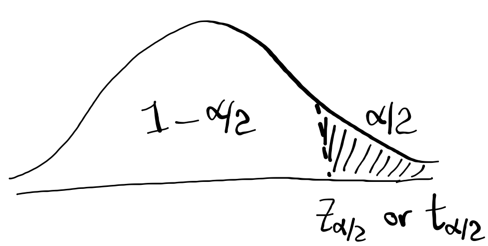
α is called Significance level

$Z_{\alpha/2}$ or $t_{\alpha/2}$ → Critical Values

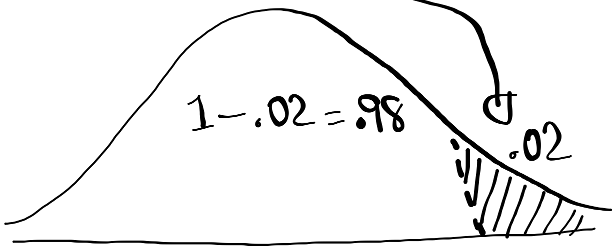
They Separate the right area $\alpha/2$ from the remaining area $1 - \alpha/2$.

we use invNorm or invT to find them.

invNorm(Left Area, 0, 1), invT(left Area, df)



Find $Z_{.02}$ $\frac{\alpha}{2} = .02 \Rightarrow \alpha = .04$

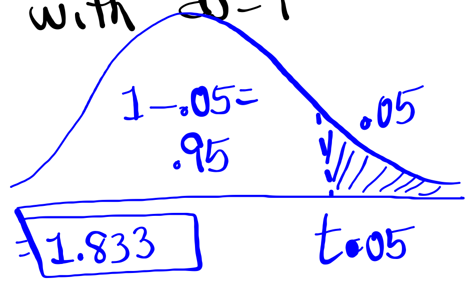


$Z_{.02} = \text{invNorm}(.98, 0, 1) = \boxed{2.054}$

Find $t_{\alpha/2}$ for $\alpha = .1$ with $df = 9$

$\frac{\alpha}{2} = \frac{.1}{2} = .05$ $t_{.05}$

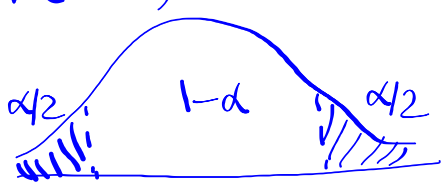
$= \text{invT}(.95, 9)$



$= \boxed{1.833}$

Confidence level ⇒ Middle Area

⇒ $(1-\alpha)\cdot 100\%$



If $\alpha = .1$ ⇒ Confidence level $= (1-.1)\cdot 100\% = 90\%$

If $\alpha = .05$ ⇒ Confidence level $= (1-.05)\cdot 100\% = 95\%$

If $\alpha = .01$ ⇒ " " $= (1-.01)\cdot 100\% = 99\%$

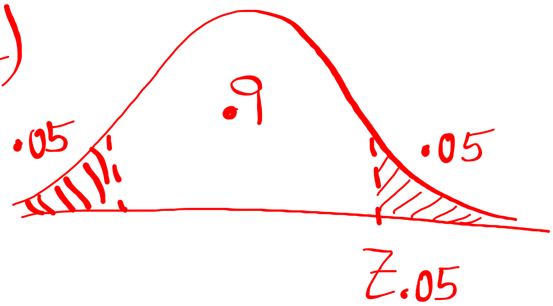
If α not given ⇒ use .05

If Confidence level not given ⇒ use .95 = 95%

Find $z_{\alpha/2}$ for 90% C-level.
(90% C-level = Middle Area)

$z_{.05} = \text{invNorm}(.95, 0, 1)$

$= \boxed{1.645}$

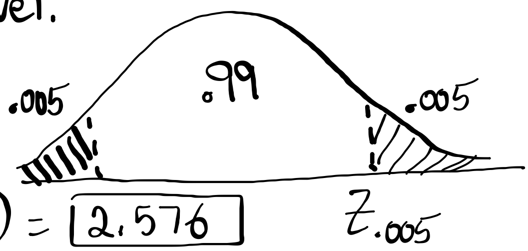


Find $z_{\alpha/2}$ for 99% C-level.

$1 - .99 = .01$

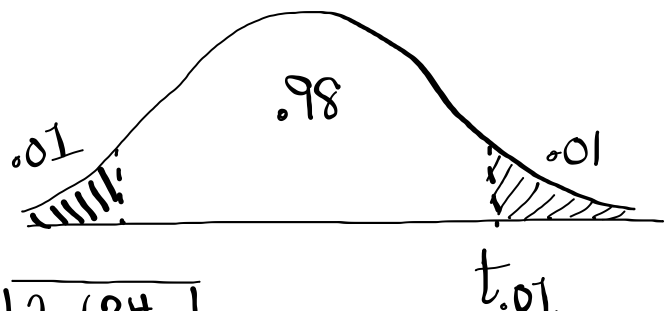
$.01 \div 2 = .005$

$z_{.005} = \text{invNorm}(.995, 0, 1) = \boxed{2.576}$



Find $t_{\alpha/2}$ for 98% C-level with df=14.

SG 22 ✓

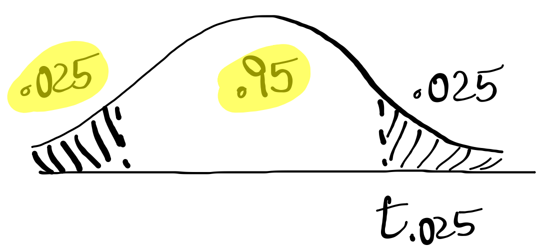


$\text{invT}(.99, 14) = \boxed{2.624}$

Find $t_{\alpha/2}$ for 95% C-level with df=49.

$\text{invT}(.975, 49)$

$= \boxed{2.010}$



Estimating Parameters:

SG 23 & 24

Population Proportion p → we use Sample Prop. $\hat{p}$ (p-hat)

Population Mean μ → we use Sample Mean $\bar{x}$ (x-bar)

Population Standard deviation σ → we use s.

When estimating Parameters, we get range of Values — Confidence Interval

For Pop. Proportion ⇒ $< p <$

For Pop. Mean ⇒ $< \mu <$

For Pop. Stand. dev. ⇒ $< \sigma <$

To build Conf. interval, we need Confidence level.

NO C-level ⇒ use .95 = 95%

Constructing Confidence Interval for Population Proportion P:

Final Ans ⇒ $< P <$

General Format ⇒ $\hat{p} - E < P < \hat{p} + E$

$\hat{p}$ Sample Proportion
Point-estimate

$\hat{p} = \frac{x}{n}$ ← x: # favorable replies; n: sample size

$\hat{q} = 1 - \hat{p}$

E Margin of error $E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$Z_{\alpha/2}$: Critical value for $(1-\alpha) \cdot 100\%$ C-level.

STAT
TESTS
1-PropZInt

Given $n = 400$ $x = 300$ C-level: .9

Find Confidence interval for pop. prop. P.

STAT
TESTS
1-PropZInt

$.714 < P < .786$

$E = \frac{.786 - .714}{2} = .036$

$\hat{p} = \frac{.786 + .714}{2} = .75$

We are 90% confident that the Pop. Prop. is between 71% & 79%.

In a Survey of 320 Students, 275 of them had iPhone.

$n = 320$

$x = 275$

Find 98% Confidence interval for the prop. of all Students that have iPhone.

→ C-level: .98

1-PropZInt

$x = 275$

$n = 320$

C-level: .98

$.814 < p < .905$

$E = \frac{.905 - .814}{2} = .046$

$\hat{p} = \frac{.905 + .814}{2} = .860$

We are 98% Confident that between 81% to 91% of all Students have iPhone.

I Surveyed 250 Students, and 6% of them were left-handed.

$\hat{p} = .06$

$n = 250$ $\hat{p} = .06$

$x = n\hat{p} = 250(.06) = 15$

if decimal → Round-up

Find Confidence interval for the prop of all Students that are left-handed.

→ NO C-level

→ use .95

1-PropZInt

$.031 < p < .089$

$E = \frac{.089 - .031}{2} = .029$

$\hat{p} = \frac{.089 + .031}{2} = .06$

We are 95% Confident that between 3% and 9% of all Students are left-handed.

Constructing Confidence Interval for Population mean μ:

Final Ans: $< \mu <$

General Format: $\bar{x} - E < \mu < \bar{x} + E$

$\bar{x}$ Sample Mean, Point-estimate,

E Margin of error

Case I : σ Known

$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

STAT Z Interval

TESTS inpt: Stats

Given: $n = 35$ $\bar{x} = 125$ $\sigma = 15$

C-level: .9

Find confidence interval for pop. mean μ.

σ Known $\Rightarrow$ ZInterval

inpt: Stats

$\sigma = 15$

$\bar{x} = 125$

$n = 35$

C-level: .9

Calculate

$120.83 < \mu < 129.17$

Since $\bar{x}$ is a whole #

Round to whole #

$121 < \mu < 129$

We are 90% Confident that the mean of all is between 121 & 129.

$E = \frac{129 - 121}{2} = 4$

$\bar{x} = \frac{129 + 121}{2} = 125$

Lisa Surveyed 25 nurses, and discovered their (mean) monthly Salary was \$6250.

$n = 25$ $\bar{x} = 6250$

It is Known that Standard deviation of Salaries of all nurses is \$400. $\sigma = 400$

Find 99% Confidence interval for the mean Salary of all nurses.

→ C-level: .99

σ Known ⇒ Z Interval

$6043.9 < \mu < 6456.1$

$\boxed{6044 < \mu < 6456}$

$E = \dfrac{6456 - 6044}{2} = \boxed{206}$

$\bar{x} = \dfrac{6456 + 6044}{2} = \boxed{6250}$

Constructing Confidence Interval for Population mean μ:

Final Ans: $\quad < \mu <$

General Format: $\bar{x} - E < \mu < \bar{x} + E$

$\bar{x}$ Sample Mean, Point-estimate,

E Margin of error

Case I: σ Known	Case II: σ Unknown
$E = Z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}$	$E = t_{\alpha/2} \cdot \dfrac{s}{\sqrt{n}}$ → $df = n-1$
STAT TESTS Z Interval inpt: Stats	STAT TESTS T Interval inpt: Stats

Given: $n=18$ $\bar{x}=30.5$ $S=8.2$

C-level: .98 → $df=18-1=17$

Find Confidence interval for Pop. mean μ.

σ Unknown ⇒ T Interval

$25.539 < \mu < 35.461$

Since $\bar{x}$ is in 1-decimal
Round to 1-decimal

$25.5 < \mu < 35.5$

$E = \frac{35.5 - 25.5}{2} = 5$

$\bar{x} = \frac{35.5 + 25.5}{2} = 30.5$

10 randomly Selected SAT exams had a mean of 1185 and Standard deviation of 100.

$n=10$

$\bar{x}=1185$ $S=100$

Find Confidence interval for the mean of all SAT exams.

→ No C-level
⇒ use .95

σ Unknown ⇒ T Interval

$1113.5 < \mu < 1256.5$

$1114 < \mu < 1257$